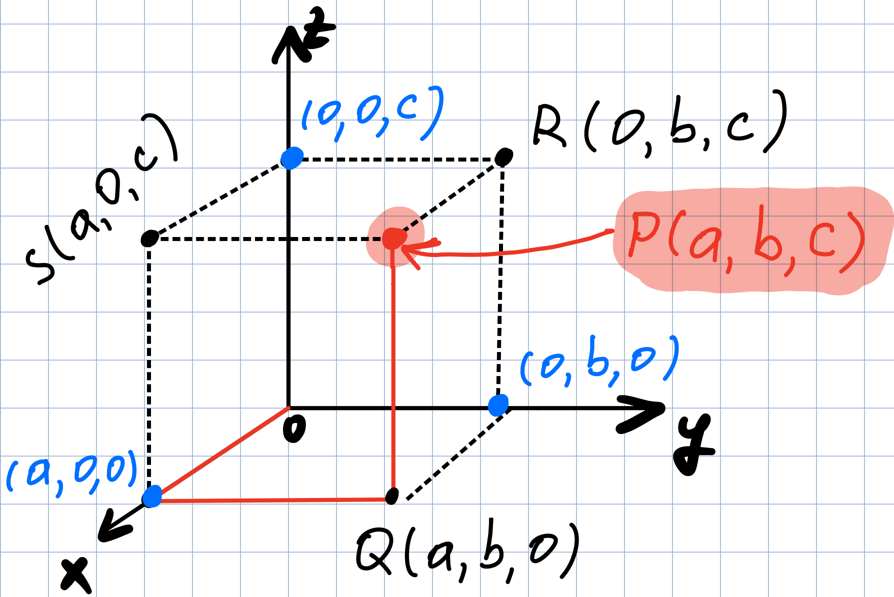
LAST TIME

The point $P(a, b, c)$ determines a rectangular box:



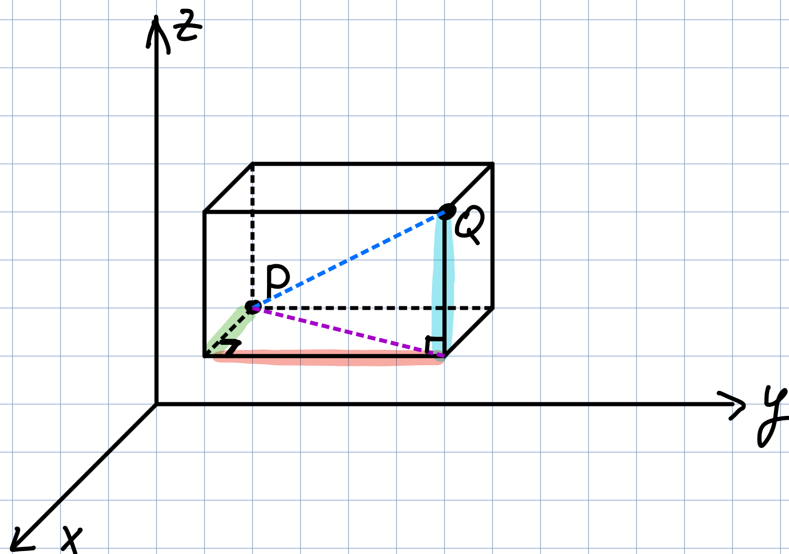
- $Q(a,b,0)$ is the projection of P onto xy-plane
- $R(0,b,c)$ is the projection of P onto yz-plane
- $S(a,0,c)$ is the projection of P onto xz-plane

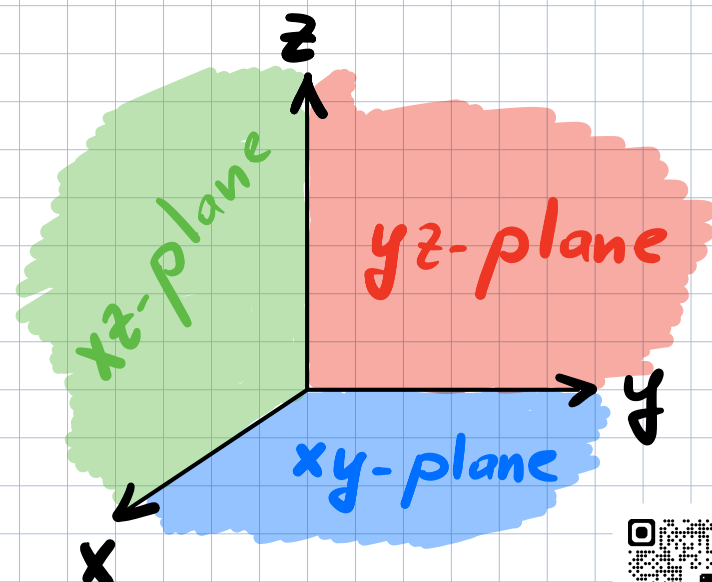
Distance:

In $\mathbb{R}^3$:

$P(x_1, y_1, z_1)$

$Q(x_2, y_2, z_2)$

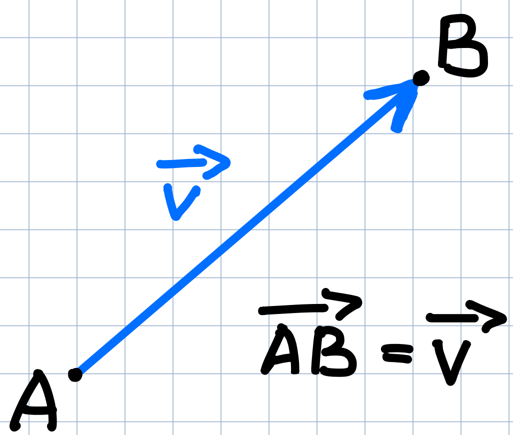


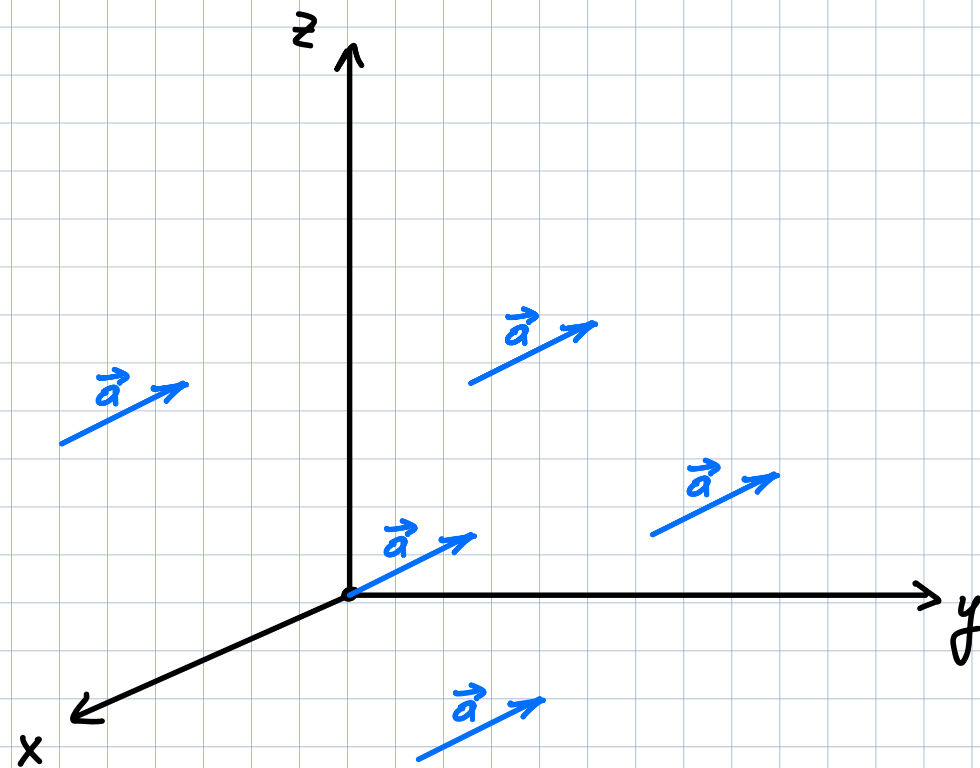


$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

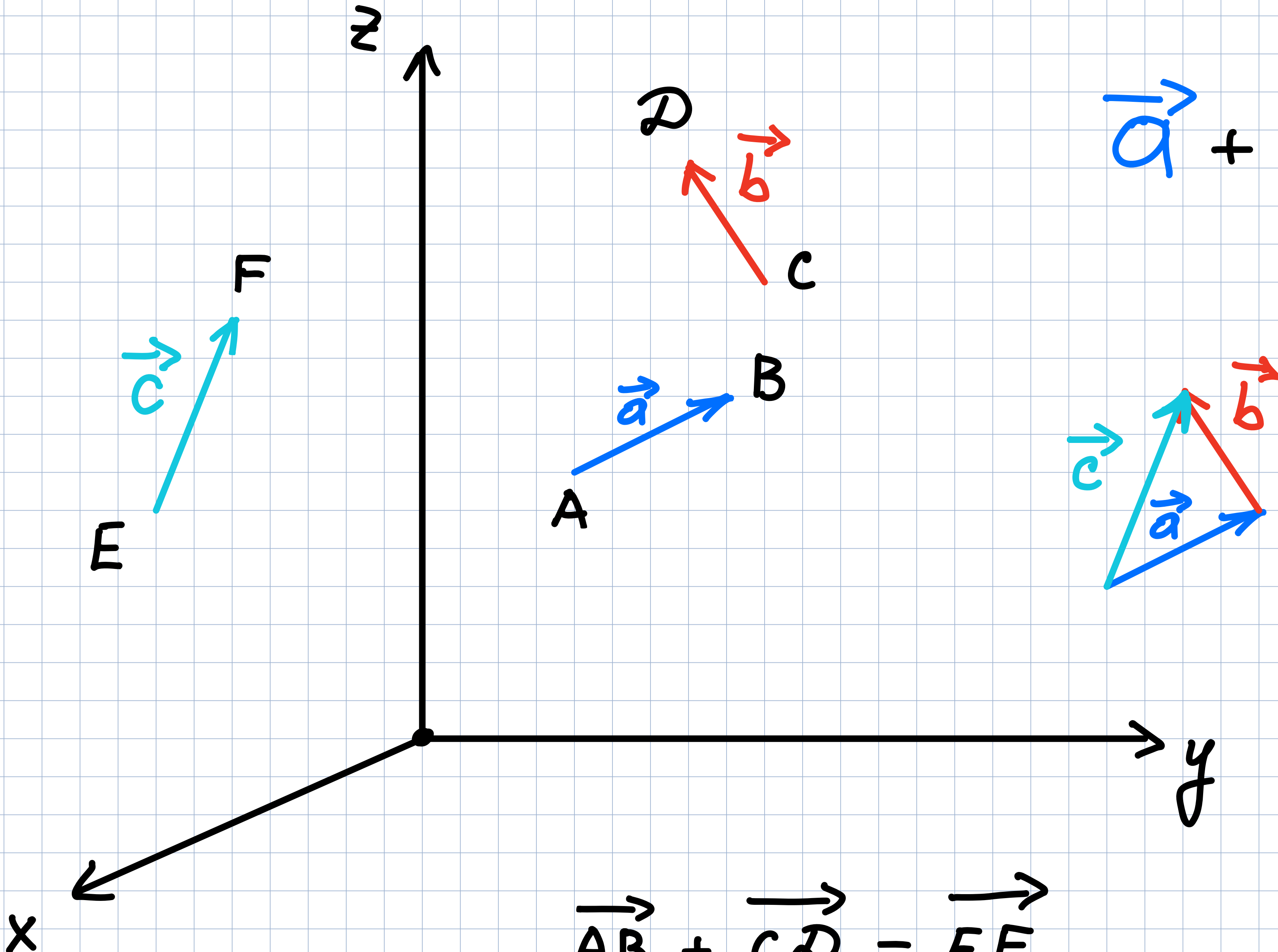
Vector

Def: A vector is a geometric object that has magnitude (or length) and direction.

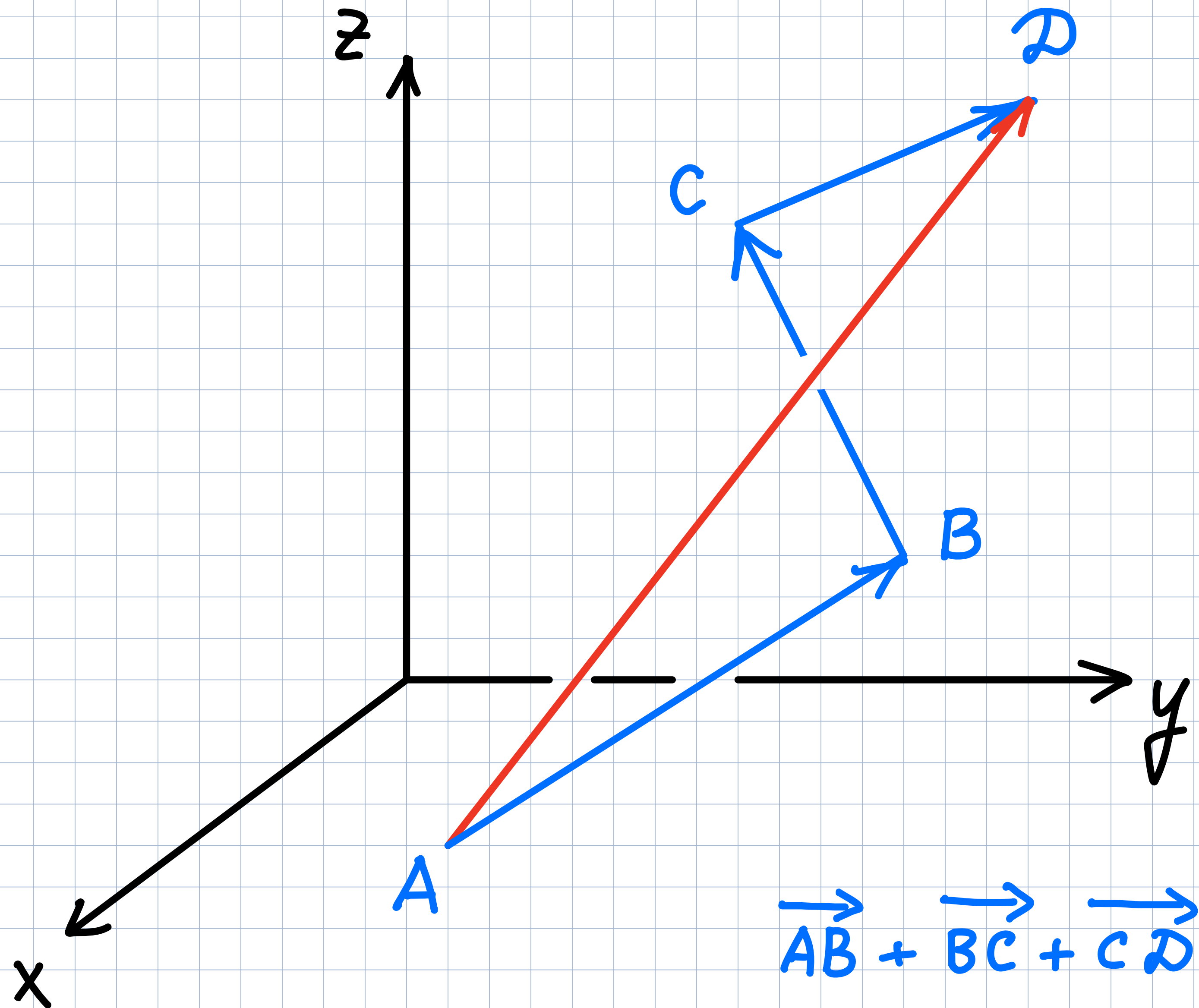




$$\vec{a} + \vec{b} = \vec{c}$$



$$\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{EF}$$

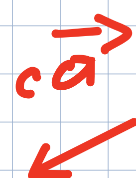


$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$$

$c \in \mathbb{R} \setminus \{0\}$

$c\vec{a}$ is a vector with magnitude $|c| \cdot |\vec{a}|$ and

$$\begin{cases} \text{the same direction as } \vec{a} & \text{if } c > 0 \\ \text{opposite direction} & \text{if } c < 0 \end{cases}$$



$$\begin{cases} c > 0 \\ |c| > 1 \end{cases} \qquad \begin{cases} c < 0 \\ |c| < 1 \end{cases} \qquad \begin{cases} c > 0 \\ |c| < 1 \end{cases}$$

Vectors

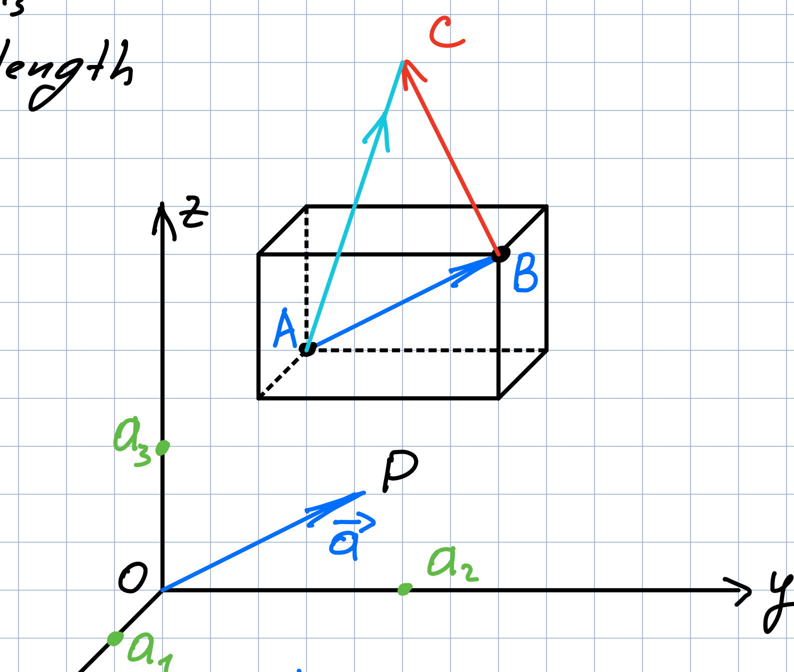
$\overrightarrow{AB} = \langle a_1, a_2, a_3 \rangle$

components

$|\overrightarrow{AB}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

magnitude or length

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



$A = (x_1, y_1, z_1) \qquad B = (x_2, y_2, z_2)$

$$a_1 = x_2 - x_1$$

$$a_2 = y_2 - y_1$$

$$a_3 = z_2 - z_1$$

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

Ex: $A(2, -3, 4) \qquad B(-2, 1, 1)$

$$\overrightarrow{AB} = \langle -2-2, 1-(-3), 1-4 \rangle = \langle -4, 4, -3 \rangle$$

$$|\overrightarrow{AB}| = \sqrt{16+16+9} = \sqrt{41}$$

Unit vector:

$$\vec{u} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{|\vec{a}|}\vec{a}$$

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

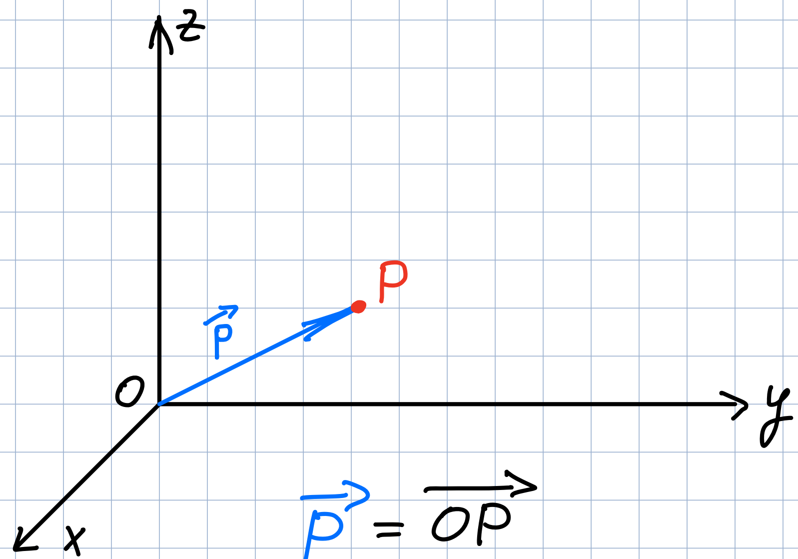
$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

For $c \in \mathbb{R}$: $c\vec{a} = \langle c a_1, c a_2, c a_3 \rangle$

$P = (a, b, c)$ - point

$\vec{p} = \langle a, b, c \rangle$ - position vector



$\vec{p} = \overrightarrow{OP}$

Def: A position vector is a vector that indicates the location of a point in space by pointing from the origin to that point.

Ex: Let $A = (2, 3, 4)$ and $B = (-1, 2, 0)$. Find position vectors $\vec{a}$ and $\vec{b}$ of points A and B and the components of the vector $\overrightarrow{BA}$.

Solution:

$\vec{a} = \overrightarrow{OA}$

$\vec{b} = \overrightarrow{OB}$

$\vec{a} = \langle 2, 3, 4 \rangle$

$\vec{b} = \langle -1, 2, 0 \rangle$

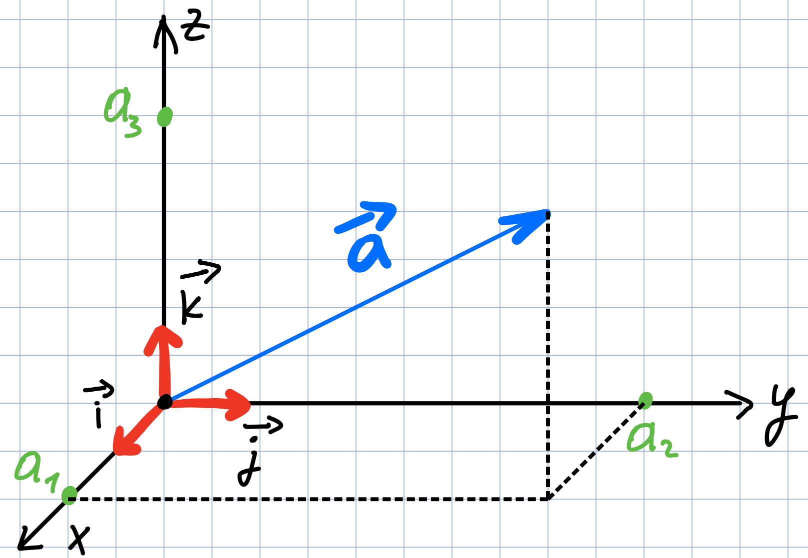
$$\overrightarrow{BA} = \langle 2-(-1), 3-2, 4-0 \rangle$$

$$= \langle 3, 1, 4 \rangle$$

components

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

Standart basis vectors



$$\vec{i} = \langle 1, 0, 0 \rangle$$

$$\vec{j} = \langle 0, 1, 0 \rangle$$

$$\vec{k} = \langle 0, 0, 1 \rangle$$

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

$$\langle a_1, a_2, a_3 \rangle = \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle$$

Ex: Find a unit vector in the direction $2\vec{i} - \vec{j} - 2\vec{k}$

Solution: $|2\vec{i} - \vec{j} - 2\vec{k}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3$

$$\frac{1}{3}(2i - j - 2k) = \frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k$$

Ex: If $\vec{a} = \langle 4, 0, 3 \rangle$ and $\vec{b} = \langle -2, 1, 5 \rangle$, find $|\vec{a}|$ and the vectors $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$, and $2\vec{a} + 5\vec{b}$.

Solution:

$$|\vec{a}| = \sqrt{4^2 + 0^2 + 3^2} = \sqrt{25} = 5$$

$$\vec{a} + \vec{b} = \langle 4 + (-2), 0 + 1, 3 + 5 \rangle = \langle 2, 1, 8 \rangle$$

$$\vec{a} - \vec{b} = \langle 4 - (-2), 0 - 1, 3 - 5 \rangle = \langle 6, -1, -2 \rangle$$

$$2\vec{a} + 5\vec{b} = 2\langle 4, 0, 3 \rangle + 5\langle -2, 1, 5 \rangle$$

$$= \langle 2\cdot 4 + 5(-2), 2\cdot 0 + 5\cdot 1, 2\cdot 3 + 5\cdot 5 \rangle$$

$$= \langle -2, 5, 31 \rangle$$

Ex: If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 7\vec{k}$, express the vector $2\vec{a} + 3\vec{b}$ in terms of $\vec{i}, \vec{j}, \vec{k}$.

Solution:

$$2\vec{a} + 3\vec{b} = 2(\vec{i} + 2\vec{j} - 3\vec{k}) + 3(4\vec{i} + 7\vec{k})$$

$$= 2\vec{i} + 4\vec{j} - 6\vec{k} + 12\vec{i} + 21\vec{k}$$

$$= 14\vec{i} + 4\vec{j} + 15\vec{k}$$